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EFDC Technical Memorandum

Theoretical and Computational Aspects of Sediment Transport in the EFDC Model

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1. Introduction

This report summarizes theoretical and computational aspects of the sediment transport formulations used in the EFDC model. Theoretical and computational aspects for the basic EFDC hydrodynamic and generic transport model components are presented in Hamrick (1992). Theoretical and computational aspects of the EFDC water quality-eutrophication model component are presented in Park et al. (1995). The paper by Hamrick and Wu (1997) also summarized computational aspects of the hydrodynamic, generic transport and water quality-eutrophication components of the EFDC model. This report is organized as follows. Chapter 2 summarizes the hydrodynamic and generic transport formulations used in EFDC. Chapter 3 summarizes the solution of the transport equation for suspended cohesive and noncohesive sediment. A discussion of near bed turbulence closure approximations relevant to sediment transport processes is present in Chapter 4. Chapters 5 and 6 summarize cohesive and noncohesive sediment settling, deposition and resuspension process representations used the sediment transport model component. The representation of the sediment bed and it geomechanical properties are presented in Chapter 7. This report will be subsequently revised to incorporate documentation of the EFDC model's sorptive contaminant transport and fate formulations as well as additional enhancements to the sediment transport formulations which are currently being tested.

2. Summary of Hydrodynamic and Generic Transport Formulations

The EFDC model's hydrodynamic component is based on the three-dimensional hydrostatic equations formulated in curvilinear-orthogonal horizontal coordinates and a sigma or stretched vertical coordinate. The momentum equations are:

$$
\begin{aligned}
&\partial_t\left(m_x m_y H u\right)+\partial_x\left(m_y H u u\right)+\partial_y\left(m_x H v u\right)+\partial_z\left(m_x m_y w u\right)-f_e m_x m_y H v \\
&=-m_y H \partial_x\left(p+p_{atm}+\phi\right)+m_y\left(\partial_x z_b^*+z \partial_x H\right)\partial_z p+\partial_z\left(m_x m_y \frac{A_v}{H} \partial_z u\right) \\
&+\partial_x\left(\frac{m_y}{m_x} H A_H \partial_x u\right)+\partial_y\left(\frac{m_x}{m_y} H A_H \partial_y u\right)-m_x m_y c_p D_p\left(u^2+v^2\right)^{1/2} u
\end{aligned}
\tag{2.1}
$$

$$
\begin{aligned}
&\partial_t\left(m_x m_y H v\right)+\partial_x\left(m_y H u v\right)+\partial_y\left(m_x H v v\right)+\partial_z\left(m_x m_y w v\right)+f_e m_x m_y H u \\
&=-m_x H \partial_y\left(p+p_{atm}+\phi\right)+m_x\left(\partial_y z_b^*+z \partial_y H\right)\partial_z p+\partial_z\left(m_x m_y \frac{A_v}{H} \partial_z v\right) \\
&+\partial_x\left(\frac{m_y}{m_x} H A_H \partial_x v\right)+\partial_y\left(\frac{m_x}{m_y} H A_H \partial_y v\right)-m_x m_y c_p D_p\left(u^2+v^2\right)^{1/2} v
\end{aligned}
\tag{2.2}
$$

$$m_x m_y f_e = m_x m_y f - u\partial_y m_x + v\partial_x m_y \tag{2.3}$$

$$(\tau_{xz}, \tau_{yz}) = A_v H^{-1}\partial_z (u, v) \tag{2.4}$$

where u and u are the horizontal velocity components in the dimensionless curvilinear-orthogonal horizontal coordinates x and y, respectively. The scale factors of the horizontal coordinates are m_x and m_y. The vertical velocity in the stretched vertical coordinate z is w. The physical vertical coordinates of the free surface and bottom bed are z_s* and z_b* respectively. The total water column depth is H, and ϕ is the free surface potential which is equal to gz_s*. The effective Coriolis acceleration f_e incorporates the curvature acceleration terms, with the Coriolis parameter, f, according to (2.3). The Q terms in (2.1) and (2.2) represent optional horizontal momentum diffusion terms. The vertical turbulent viscosity A_v relates the shear stresses to the vertical shear of the horizontal velocity components by (4.4). The kinematic atmospheric pressure, referenced to water density, is p_{atm}, while the excess hydrostatic pressure in the water column is given by:

$$\partial_z p = -gHb = -gH(\rho - \rho_o)\rho_o^{-1} \tag{2.5}$$

where ρ and ρ_o are the actual and reference water densities and b is the buoyancy. The horizontal turbulent stress on the last lines of (2.1) and (2.2), with A_H being the horizontal turbulent viscosity, are typically retained when the advective acceleration are represented by central differences. The last terms in (2.1) and (2.2) represent vegetation resistance where c_p is a resistance coefficient and D_p is the dimensionless projected vegetation area normal to the flow per unit horizontal area.

The three-dimensional continuity equation in the stretched vertical and curvilinear-orthogonal horizontal coordinate system is:

$$\partial_t (m_x m_y H) + \partial_x (m_y H u) + \partial_y (m_x H v) + \partial_z (m_x m_y w) = Q_H \tag{2.6}$$

with Q_H representing volume sources and sinks including rainfall, evaporation, infiltration and lateral inflows and outflows having negligible momentum fluxes. When the sediment transport model component operates in a geomorphologic mode, Q_H also includes the volume flux of sediment and water between the sediment bed and the water column. Integration of (2.6) over the water column gives

$$\partial_t (m_x m_y H) + \partial_x (m_y H \bar{u}) + \partial_y (m_x H \bar{v}) = \bar{Q}_H \tag{2.7}$$

the barotropic or external mode continuity equation where the over bars indicate depth averaged quantities. Subtracting (2.7) form (2.6) gives

$$\partial_x (m_y H(u - \bar{u})) + \partial_y (m_x H(v - \bar{v})) + \partial_z (m_x m_y w) = Q_H - \bar{Q}_H \tag{2.8}$$

the internal mode continuity equation.

The generic transport equation for a dissolved or suspended material having a mass per unit volume concentration *C*, is

$$\begin{aligned}&\partial_t\left(m_x m_y HC\right)+\partial_x\left(m_y HuC\right)+\partial_y\left(m_x HvC\right)+\partial_z\left(m_x m_y wC\right)-\partial_z\left(m_x m_y w_{sc} C\right)\\&\quad=\partial_x\left(\frac{m_y}{m_x}HK_H\partial_x C\right)+\partial_y\left(\frac{m_x}{m_y}HK_H\partial_y v\right)+\partial_z\left(m_x m_y\frac{K_v}{H}\partial_z C\right)+Q_c\end{aligned}\tag{2.9}$$

where K_v and K_H are the vertical and horizontal turbulent diffusion coefficients, respectively, w_{sc} is a positive settling velocity went *C* represents a suspended material, and Q_c represents external sources and sinks and reactive internal sources and sinks.

The solution of the momentum equations, (2.1) and (2.2) and the transport equation (2.9), requires the specification of the vertical turbulent viscosity, A_v, and diffusivity, K_v. To provide the vertical turbulent viscosity and diffusivity, the second moment turbulence closure model developed by Mellor and Yamada (1982) and modified by Galperin *et al* (1988) and Blumberg *et al* (1988) is used. The MY model relates the vertical turbulent viscosity and diffusivity to the turbulent intensity, *q*, a turbulent length scale, *l*, and a turbulent intensity and length scaled based Richardson number, R_q, by:

$$\begin{aligned}&A_v=\phi_A ql\\&\phi_A=\frac{A_o\left(1+R_1^{-1}R_q\right)}{\left(1+R_2^{-1}R_q\right)\left(1+R_3^{-1}R_q\right)}\\&A_o=A_1\left(1-3C_1-\frac{6A_1}{B_1}\right)=\frac{1}{B_1^{1/3}}\\&R_1^{-1}=3A_2\frac{\left(B_2-3A_2\right)\left(1-\frac{6A_1}{B_1}\right)-3C_1\left(B_2+6A_1\right)}{\left(1-3C_1-\frac{6A_1}{B_1}\right)}\\&R_2^{-1}=9A_1A_2\\&R_3^{-1}=3A_2\left(6A_1+B_2\right)\end{aligned}\tag{2.10}$$

$$\begin{aligned}&K_v=\phi_K ql\\&\phi_K=\frac{K_o}{\left(1+R_3^{-1}R_q\right)}\\&K_o=A_2\left(1-\frac{6A_1}{B_1}\right)\end{aligned}\tag{2.11}$$

$$R_q = -\frac{gH\partial_z b}{q^2}\frac{l^2}{H^2} \tag{2.12}$$

where the so-called stability functions, ϕ_A and ϕ_K, account for reduced and enhanced vertical mixing or transport in stable and unstable vertically density stratified environments, respectively. Mellor and Yamada (1982) specify the constants A_1, B_1, C_1, A_2, and B_2 as 0.92, 16.6, 0.08, 0.74, and 10.1, respectively.

The turbulent intensity and the turbulent length scale are determined by a pair of transport equations:

$$\begin{aligned}
&\partial_t\left(m_x m_y H q^2\right) + \partial_x\left(m_y H u q^2\right) + \partial_y\left(m_x H v q^2\right) + \partial_z\left(m_x m_y w q^2\right) \\
&= \partial_z\left(m_x m_y \frac{A_q}{H}\partial_z q^2\right) - 2 m_x m_y \frac{H q^3}{B_1 l} \\
&+2 m_x m_y\left(\frac{A_v}{H}\left((\partial_z u)^2 + (\partial_z v)^2\right) + \eta_p c_p D_p\left(u^2 + v^2\right)^{3/2} + g K_v \partial_z b\right) + Q_q
\end{aligned} \tag{2.13}$$

$$\begin{aligned}
&\partial_t\left(m_x m_y H q^2 l\right) + \partial_x\left(m_y H u q^2 l\right) + \partial_y\left(m_x H v q^2 l\right) + \partial_z\left(m_x m_y w q^2 l\right) \\
&= \partial_z\left(m_x m_y \frac{A_q}{H}\partial_z\left(q^2 l\right)\right) - m_x m_y \frac{H q^3}{B_1}\left(1 + E_2\left(\frac{l}{\kappa H z}\right)^2 + E_3\left(\frac{l}{\kappa H(1-z)}\right)^2\right) \\
&+ m_x m_y E_1 l\left(\frac{A_v}{H}\left((\partial_z u)^2 + (\partial_z v)^2\right) + g K_v \partial_z b + \eta_p c_p D_p\left(u^2 + v^2\right)^{3/2}\right) + Q_l
\end{aligned} \tag{2.14}$$

where (E_1, E_2, E_3) = (1.8, 1.33, 0.25). The third on the last line of each equation represents net turbulent energy production by vegetation drag where η_p is a production efficiency factor have a value less than one. The terms Q_q and Q_l may represent additional source-sink terms such as subgrid scale horizontal turbulent diffusion. The vertical diffusivity, A_q, is set to *0.2ql* as recommended by Mellor and Yamada (1982) For stable stratification, Galperin *et al* (1988) suggest limiting the length scale such that the square root of R_q is less than 0.52. When horizontal turbulent viscosity and diffusivity are included in the momentum and transport equations, they are determined independently using Smagorinsky's (1963) subgrid scale closure formulation.

Vertical boundary conditions for the solution of the momentum equations are based on the specification of the kinematic shear stresses, equation (2.4), at the bed and the free surface. At the free surface, the *x* and *y* components of the stress are specified by the water surface wind stress

$$\left(\tau_{xz}, \tau_{yz}\right) = \left(\tau_{sx}, \tau_{sy}\right) = c_s\sqrt{U_w^2 + V_w^2}\left(U_w, V_w\right) \tag{2.15}$$

where U_w and V_w are the x and y components of the wind velocity at 10 meters above the water surface. The wind stress coefficient is given by:

$$c_s = 0.001\frac{\rho_a}{\rho_w}\left(0.8 + 0.065\sqrt{U_w^2 + V_w^2}\right) \tag{2.16}$$

for the wind velocity components in meters per second, with ρ_a and ρ_w denoting air and water densities respectively. At the bed, the stress components are presumed to be related to the near bed or bottom layer velocity components by the quadratic resistance formulation

$$(\tau_{xz}, \tau_{yz}) = (\tau_{bx}, \tau_{by}) = c_b\sqrt{u_1^2 + v_1^2}\,(u_1, v_1) \tag{2.17}$$

where the 1 subscript denotes bottom layer values. Under the assumption that the near bottom velocity profile is logarithmic at any instant of time, the bottom stress coefficient is given by

$$c_b = \left(\frac{\kappa}{\ln(\Delta_1 / 2z_o)}\right)^2 \tag{2.18}$$

where κ, is the von Karman constant, Δ_1 is the dimensionless thickness of the bottom layer, and $z_o = z_o{*}/H$ is the dimensionless roughness height. Vertical boundary conditions for the turbulent kinetic energy and length scale equations are:

$$q^2 = B_1^{2/3}|\tau_s| \quad : \quad z = 1 \tag{2.19}$$

$$q^2 = B_1^{2/3}|\tau_b| \quad : \quad z = 0 \tag{2.20}$$

$$l = 0 \quad : \quad z = 0,1 \tag{2.21}$$

where the absolute values indicate the magnitude of the enclosed vector quantity. Equations (2.17) and (2.18) can become inappropriate under a number of conditions associated with either or both high near bottom sediment concentrations and high frequency surface wave activity. The quantification of sediment and wave effects on the bottom stress is discussed in Chapter 4.

3. Solution of the Sediment Transport Equation

The EFDC model uses a high order upwind difference solution scheme for the advective terms in the transport equation. Although the scheme is designed to minimize numerical diffusion, a small amount of horizontal diffusion remains inherent in the scheme. Due the small inherent numerical diffusion, the physical horizontal diffusion terms in (2.9) are omitted as to give:

$$
\begin{aligned}
&\partial_t\left(m_x m_y H S_j\right)+\partial_x\left(m_y H u S_j\right)+\partial_y\left(m_x H v S_j\right)+\partial_z\left(m_x m_y w S_j\right) \\
&\quad -\partial_z\left(m_x m_y w_{sj} S_j\right)=\partial_z\left(m_x m_y \frac{K_V}{H}\partial_z S_j\right)+Q_{sj}^E+Q_{sj}^I
\end{aligned}
\tag{3.1}
$$

where S_j represents the concentration of the jth sediment class and the source-sink term has been split into an external part, which would include point and nonpoint source loads, and internal part which could include reactive decay of organic sediments or the exchange of mass between sediment classes if floc formation and destruction were simulated. Vertical boundary conditions for (3.1) are:

$$
\begin{aligned}
&-\frac{K_V}{H}\partial_z S_j - w_s S = J_{jo} : z \approx 0 \\
&-\frac{K_V}{H}\partial_z S_j - w_{sj} S_j = 0 : z = 1
\end{aligned}
\tag{3.2}
$$

where J_o is the net water column-bed exchange flux defined as positive into the water column.

The numerical solution of (3.1) utilizes a fractional step procedure. The first step advances the concentration due to advection and external sources and sinks having corresponding volume fluxes by

$$
\begin{aligned}
&H^{n+1}S^* = H^n S^n + \frac{\theta}{m_x m_y}\left(Q_{sj}^E\right)^{n+1/2} \\
&-\frac{\theta}{m_x m_y}\left(\partial_x\left(m_y (Hu)^{n+1/2} S^n\right)+\partial_y\left(m_x (Hv)^{n+1/2} S^n\right)+\partial_z\left(m_x m_y w^{n+1/2} S^n\right)\right)
\end{aligned}
\tag{3.3}
$$

where n and $n+1$ denote the old and new time levels and * denotes the intermediate fractional step results. The portion of the source and sink term, associated with volumetric sources and sinks is included in the advective step for consistency with the continuity constraint. This term, as well as the advective field (u,v,w), is defined as intermediate in time between the old and new time levels consistent with continuity. Note that the sediment class subscripts have be dropped for clarity. The advection set uses the antidiffusive MPDATA scheme (Smolarkiewicz and Clark, 1986) with optional flux corrected transport (Smolarkiewicz and Grabowski, 1990).

The second fractional settling step is given by

$$
S^{**} = S^* + \frac{\theta}{H^{n+1}}\partial_z\left(w_s S^{**}\right)
\tag{3.4}
$$

which is solved by a fully implicit upwind difference scheme with an optional antidiffusion correction across internal water column layer interfaces. For the bottom bed adjacent layer, (3.4) is written as:

$$S_1^{**} = S_1^{*} + \frac{\theta}{\Delta_1 H^{n+1}} \left(w_s S^{**} \right)_2 - \frac{\theta}{\Delta_z H^{n+1}} \left(w_s S^{**} \right)_1 \tag{3.5}$$

The water column-bed flux (3.2) can be written as

$$-\frac{K_V}{H} \partial_z S_j - w_s S = J_o = w_r S_r - P_d w_s S \tag{3.6}$$

where the product, $w_r S_r$ symbolically represents the resuspension flux and P_d the probability of deposition which is less than or equal to one. Since the remaining step will represent diffusion, for solution efficiency, the diffusive flux at the bed in (3.6) is set to zero in the settling and subsequent diffusion set. Equation (3.5) then becomes

$$\left(1 + \frac{\theta P_d w_s}{\Delta_z H^{n+1}} \right) S_1^{**} = S_1^{*} + \frac{\theta}{\Delta_1 H^{n+1}} \left(w_s S^{**} \right)_2 + \frac{\theta}{\Delta_z H^{n+1}} w_r S_r \tag{3.7}$$

In the actual EFDC code, if the net bed flux, J_o is positive, it is limited such that only the current top layer of the bed can be completely resuspended in single time step. The remaining fractional step is an implicit diffusion step

$$S^{n+1} = S^{**} + \theta \partial_z \left(\left(\frac{K_V}{H^2} \right)^{n+1} \partial_z S^{n+1} \right) \tag{3.8}$$

with zero diffusive fluxes at the bed and water surface.

4. Near Bed Turbulence Closure

The proper formulation of hydrodynamic and sediment boundary layer parameterization appropriate for representing the bottom stress and the water column-bed exchange of sediment under conditions including high frequency surface waves and high near bed suspended sediment gradients should be based upon the near bed turbulent kinetic energy balance. The near bed balance assumes an equilibrium between production of turbulence by shear stresses, vegetation drag, and unstable density stratification, the suppression of turbulence by stable stratification, and the dissipation. The turbulent kinetic energy equation (2.13) reduces to

$$\frac{A_v}{H} \left(\left(\partial_z u \right)^2 + \left(\partial_z v \right)^2 \right) + c_p \left(u^2 + v^2 \right)^{3/2} + g K_v \partial_z b = \frac{H q^3}{B_1 l} \tag{4.1}$$

Multiplying (4.1) by A_v/H and using (2.4) gives

$$\left(\tau_{xz}^2+\tau_{yz}^2\right)+c_p\frac{A_v}{H}\left(u^2+v^2\right)^{3/2}+gK_v\frac{A_v}{H}\partial_z b=\frac{A_v}{H}\frac{Hq^3}{B_1 l} \tag{4.2}$$

In the absence of vegetation and stratification, evaluation of (4.2) at the bed, using (2.10) gives

$$\left(\tau_{xz}^2+\tau_{yz}^2\right)_b=\left|\tau_b\right|^2=\frac{1}{B_1^{1/3}}q\frac{l}{H}\frac{Hq^3}{B_1 l}=\frac{q^4}{B_1^{4/3}} \tag{4.3}$$

recovering the boundary condition (2.20). For the general case, the definition of A_v is introduced into (4.2) to give

$$q^4-B_1\left(gH\frac{l}{H}\frac{K_v}{H}\partial_z b+c_p\frac{l}{H}\left(u^2+v^2\right)^{3/2}\right)q-\frac{B_1}{\phi_A}\left(\tau_{xz}^2+\tau_{yz}^2\right)=0 \tag{4.4}$$

Near the bed for three-dimensional model applications and over the depth for two-dimensional applications, the turbulent length scale can be specified by the algebraic relationship

$$\frac{l}{H}=\kappa z(1-z)^{\lambda} \tag{4.5}$$

If high frequency surface waves are present, the shear stress can be decomposed into current and wave components

$$\begin{aligned}\tau_{xz}&=\tau_c\cos\psi_c+\tau_w\cos\psi_w\\ \tau_{yz}&=\tau_c\sin\psi_c+\tau_w\sin\psi_w\end{aligned} \tag{4.6}$$

where τ_c and τ_w are the current and wave shear stress magnitudes. Evaluating the stress term in (4.4) gives

$$\left(\tau_{xz}^2+\tau_{yz}^2\right)=\tau_c^2+\tau_w^2+2\left(\cos\psi_c\cos\psi_w+\sin\psi_c\sin\psi_w\right)\tau_c\tau_w \tag{4.7}$$

Assuming the wave shear stress to be periodic

$$\tau_w=\tau_{wm}\sin(\omega t) \tag{4.8}$$

the mean square stress average over the wave period is given by

$$\left\langle\tau_{xz}^2+\tau_{yz}^2\right\rangle=\tau_c^2+\frac{1}{2}\tau_{wm}^2 \tag{4.9}$$

For wave periods much smaller than the time step of the numerical integration, (4.4) is well approximated using (4.9) as

$$q^4 - B_1\left(gH\frac{l}{H}\frac{K_v}{H}\partial_z b + c_p \frac{l}{H}\left(u^2+v^2\right)^{3/2}\right) q - \frac{B_1}{\phi_A}\left(\tau_c^2 + \frac{1}{2}\tau_{wm}^2\right) = 0 \qquad (4.10)$$

The buoyancy gradient near the bed is primarily due to gradients in suspended sediment concentration with the effect of sediment on density given by

$$\rho = \left(\frac{\varepsilon}{1+\varepsilon}\right)\rho_w + \left(\frac{1}{1+\varepsilon}\right)\rho_s = \left(\frac{\varepsilon}{1+\varepsilon}\right)\rho_w + S \qquad (4.11)$$

where ε is the void ratio of the sediment water mixture and S is the mass concentration of sediment. The buoyancy can be expressed in terms of the sediment concentration using

$$b = \frac{\rho - \rho_w}{\rho_w} = \left(\frac{\rho_s - \rho_w}{\rho_w \rho_s}\right) S = \alpha S \qquad (4.12)$$

with (4.10) becoming

$$q^4 - B_1\left(\alpha gH\frac{l}{H}\frac{K_v}{H}\partial_z S + c_p \frac{l}{H}\left(u^2+v^2\right)^{3/2}\right) q - \frac{B_1}{\phi_A}\left(\tau_c^2 + \frac{1}{2}\tau_{wm}^2\right) = 0 \qquad (4.13)$$

Equation (4.13) provides an algebraic equation for specifying the turbulent intensity q at any level in the hydrodynamic and sediment boundary layers. Since the boundary layer parameter are recalculated at each time step of the hydrodynamic model integration, the solution of (4.13) can be approximated by

$$\left(q^4\right)^{n+1} = B_1\left(\alpha gH\frac{l}{H}\frac{K_v}{H}\partial_z S q + c_p \frac{l}{H}\left(u^2+v^2\right)^{3/2} q\right)^n + \frac{B_1}{\phi_A}\left(\tau_c^2 + \frac{1}{2}\tau_{wm}^2\right)^n \qquad (4.14)$$

where *n+1* and *n* denote the new and old time levels, respectively. Since the vertical gradient of the sediment concentration is generally negative, there is low possibility of the right side of (4.13) also being negative. In such and event, the turbulent intensity is set to a small value on the order of 1E-4.

5. Noncohesive Sediment Settling, Deposition and Resuspension

Noncohesive inorganic sediments settle as discrete particles, with hindered settling and multiphase interactions becoming important in regions of high sediment concentration near the bed. At low concentrations, the settling velocity for the jth noncohesive sediment class corresponds to the settling velocity of a discrete particle:

$$w_{sj} = w_{soj} \tag{5.1}$$

At higher concentrations and hindering settling conditions, the settling velocity is less than the discrete velocity and can be expressed in the form

$$w_{sj} = \left(1 - \sum_{i}^{I} \frac{S_i}{\rho_{si}}\right)^n w_{soj} \tag{5.2}$$

where ρ_s is the sediment particle density with values of n ranging from 2 (Cao et al., 1996) to 4 (Van Rijn, 1984). The expression (5.2) is approximated to within 5 per cent by

$$w_{sj} = \left(1 - n\sum_{i}^{I} \frac{S_i}{\rho_{si}}\right) w_{soj} \tag{5.3}$$

for total sediment concentrations up to 200,000 mg/liter. For total sediment concentrations less than 25,000 mg/liter, neglect of the hindered settling correction results in less than a 5 per cent error in the settling velocity, which is well within the range of uncertainty in parameters used to estimate the discrete particle settling velocity.

At the water column-sediment bed interface, the net flux of noncohesive sediment is controlled primarily by the shear stress exerted by the near bed flow and the size and density of the noncohesive material at the bed surface. Under steady, uniform flow and sediment loading conditions, an equilibrium distribution of sediment in the water column tends to be established, with the resuspension and deposition fluxes canceling each other. Using a number of simplifying assumptions, the equilibrium sediment concentration distribution in the water column can be expressed analytically in terms of the near bed reference or equilibrium concentration, the settling velocity and the vertical turbulent diffusivity. For unsteady or spatially varying flow conditions, the water column sediment concentration distribution varies in space and time in response to sediment load variations, changes in hydrodynamic transport, and associated nonzero fluxes across the water column-sediment bed interface. An increase or decrease in the bed stress and the intensity of vertical turbulent mixing will result in net erosion or deposition, respectively, at a particular location or time.

To illustrate how an appropriate sediment bed flux boundary condition can be established, consider the approximation to the sediment transport equation (3.1) for nearly uniform horizontal conditions

$$\partial_t (HS) = \partial_z \left(\frac{K_v}{H} \partial_z S + w_s S \right) \tag{5.4}$$

Integrating (5.4) over the depth of the bottom hydrodynamic model layer gives

$$\partial_t\left(\Delta H\bar{S}\right) = J_0 - J_\Delta \tag{5.5}$$

where the over bar denotes the mean over the dimensionless layer thickness, *Δ*. Subtracting (5.5) from (5.4) gives

$$\partial_t\left(HS'\right) = \partial_z\left(\frac{K_v}{H}\partial_z S + w_s S\right) - \left(\frac{J_0 - J_\Delta}{\Delta}\right) \tag{5.6}$$

Assuming that the rate of change of the deviation of the sediment concentration from the mean is small

$$\partial_t\left(HS'\right) \le \partial_t\left(H\bar{S}\right) \tag{5.7}$$

allows (5.6) to be approximated by

$$\partial_z\left(\frac{K_v}{H}\partial_z S + w_s S\right) = \left(\frac{J_0 - J_\Delta}{\Delta}\right) \tag{5.8}$$

Integrating (5.8) once gives

$$\frac{K_v}{H}\partial_z S + w_s S = \left(\frac{J_0 - J_\Delta}{\Delta}\right)\frac{z}{\Delta} - J_0 \tag{5.9}$$

Very near the bed, (5.9) can be approximated by

$$\frac{K_v}{H}\partial_z S + w_s S = -J_0 \tag{5.10}$$

Neglecting stratification effects and using the results of Chapter 4, the near bed diffusivity is approximately

$$\frac{K_v}{H} = K_o q\frac{l}{H} \cong u_* \kappa z \tag{5.11}$$

Introducing (5.11) into (5.10) gives

$$\partial_z S + \frac{R}{z}S = -\frac{R}{z}\frac{J_o}{w_s} \tag{5.12}$$

where

$$R = \frac{w_s}{u_* \kappa} \tag{5.13}$$

is the Rouse parameter. The solution of (5.12) is

$$S = -\frac{J_o}{w_s} + \frac{C}{z^R} \tag{5.14}$$

The constant of integration is evaluated using

$$S = S_{eq} \quad : \quad z = z_{eq} \quad and \quad J_o = 0 \tag{5.15}$$

which sets the sediment concentration to an equilibrium value, defined just above the bed under not net flux condition. Using (5.15), equation (5.14) becomes

$$S = \left(\frac{z_{eq}}{z}\right)^R S_{eq} - \frac{J_o}{w_s} \tag{5.16}$$

For nonequilibrium conditions, the net flux is given by evaluating (5.16) at the equilibrium level

$$J_o = w_s \left(S_{eq} - S_{ne} \right) \tag{5.17}$$

where S_{ne} is the actual concentration at the reference equilibrium level.

Equation (5.17) clearly indicates that when the near bed sediment concentration is less than the equilibrium value a net flux from the bed into the water column occurs. Likewise when the concentration exceeds equilibrium, a net flux to the bed occurs. For the relationship (5.17) to be useful in a numerical model, the bed flux must be expressed in terms of the layer mean concentration. For a three-dimensional application, (5.16) can be integrated over the bottom model layer to give

$$J_o = w_s \left(\bar{S}_{eq} - \bar{S} \right) \tag{5.18}$$

where

$$\begin{aligned} \bar{S}_{eq} &= \frac{\ln\left(\Delta z_{eq}^{-1}\right)}{\left(\Delta z_{eq}^{-1} - 1\right)} S_{eq} : R = 1 \\ \bar{S}_{eq} &= \frac{\left(\left(\Delta z_{eq}^{-1}\right)^{1-R} - 1\right)}{(1-R)\left(\Delta z_{eq}^{-1} - 1\right)} S_{eq} : R \neq 1 \end{aligned} \tag{5.19}$$

defines an equivalent layer mean equilibrium concentration in terms of the near bed equilibrium concentration. The corresponding quantities in the numerical solution bottom boundary condition (3.6) are

$$w_r S_r = w_s \bar{S}_{eq} \qquad (5.20)$$
$$P_d w_s = w_s$$

If the dimensionless equilibrium elevation, z_{eq} exceeds the dimensionless layer thickness, (5.19) can be modified to

$$\bar{S}_{eq} = \frac{\ln\left(M\Delta z_{eq}^{-1}\right)}{\left(M\Delta z_{eq}^{-1} - 1\right)} S_{eq} : R = 1$$
$$\bar{S}_{eq} = \frac{\left(\left(M\Delta z_{eq}^{-1}\right)^{1-R} - 1\right)}{(1-R)\left(M\Delta z_{eq}^{-1} - 1\right)} S_{eq} : R \neq 1 \qquad (5.21)$$

where the over bars in (5.18) and (5.20) implying an average of the first M layers above the bed.

For two-dimensional, depth averaged model application, a number of additional consideration are necessary. For depth average modeling, the equivalent of (5.9) is

$$\frac{K_v}{H} \partial_z S + w_s S = -J_o(1 - z) \qquad (5.22)$$

Neglecting stratification effects and using the results of Chapter 4, the diffusivity is

$$\frac{K_v}{H} = K_o q \frac{l}{H} \cong u_* \kappa z (1 - z)^{\lambda} \qquad (5.23)$$

Introducing (5.23) into (5.22) gives

$$\partial_z S + \frac{R}{z(1-z)^{\lambda}} S = -\frac{R(1-z)^{1-\lambda}}{z} \frac{J_o}{w_s} \qquad (5.24)$$

A close form solution of (5.24) is possible for λ equal to zero. Although the resulting diffusivity is not as reasonable as the choice of λ equal to one, the resulting vertical distribution of sediment is much more sensitive to the near bed diffusivity distribution than the distribution in the upper portions of the water column. For λ equal to zero, the solution of (5.23) is

$$S = -\left(1 - \frac{Rz}{(1+R)}\right) \frac{J_o}{w_s} + \frac{C}{z^R} \qquad (5.25)$$

Evaluating the constant of integration using (5.15) gives

$$S = \left(\frac{z_{eq}}{z}\right)^R S_{eq} - \left(1 - \frac{Rz}{(1+R)}\right)\frac{J_o}{w_s} \tag{5.26}$$

For nonequilibrium conditions, the net flux is given by evaluating (5.26) at the equilibrium level

$$J_o = w_s\left(\frac{(1+R)}{1+R(1-z_{eq})}\right)(S_{eq} - S_{ne}) \tag{5.27}$$

where S_{ne} is the actual concentration at the reference equilibrium level. Since z_{eq} is on the order of the sediment grain diameter divided by the depth of the water column, (5.27) is essentially equivalent (5.17). To obtain an expression for the bed flux in terms of the depth average sediment concentration, (5.26) is integrated over the depth to give

$$J_o = w_s\left(\frac{2(1+R)}{2+R(1-z_{eq})}\right)(\bar{S}_{eq} - \bar{S}) \tag{5.28}$$

where

$$\begin{aligned} \bar{S}_{eq} &= \frac{\ln(z_{eq}^{-1})}{(z_{eq}^{-1}-1)} S_{eq} \; : \; R = 1 \\ \bar{S}_{eq} &= \frac{(z_{eq}^{R-1}-1)}{(1-R)(z_{eq}^{-1}-1)} S_{eq} \; : R \neq 1 \end{aligned} \tag{5.29}$$

The corresponding quantities in the numerical solution bottom boundary condition (3.6) are

$$\begin{aligned} w_r S_r &= w_s\left(\frac{2(1+R)}{2+R(1-z_{eq})}\right)\bar{S}_{eq} \\ P_d w_s &= \left(\frac{2(1+R)}{2+R(1-z_{eq})}\right) w_s \end{aligned} \tag{5.30}$$

When multiple sediment size classes are simulated, the equilibrium concentrations given by (5.19), (5.21), and (5.29) are adjusted by multiplying by their respective sediment volume fractions in the surface layer of the bed.

The specification of the water column-bed flux of noncohesive sediment has been reduced to specification of the near bed equilibrium concentration and it corresponding reference distance above the bed. Garcia and Parker (1991) evaluated seven relationships, based on combinations of analysis and experiment correlation, for determining the near bed equilibrium concentration as well as proposing a new relationship. All of the relationships essential specify the equilibrium concentration in terms of hydrodynamic and sediment physical parameters

$$S_{eq} = S_{eq}(d, \rho_s, \rho_w, w_s, u_*, \nu) \tag{5.31}$$

including the sediment particle diameter, the sediment and water densities, the sediment settling velocity, the bed shear velocity, and the kinematic molecular viscosity of water. Garcia and Parker concluded that the representations of Smith and McLean (1977) and Van Rijn (1984) as well as their own proposed representation perform acceptably when tested against experimental and field observations.

Smith and McLean's formula for the equilibrium concentration is

$$S_{eq} = \rho_s \frac{0.65\gamma_o T}{1 + \gamma_o T} \tag{5.32}$$

where γ_o is a constant equal to 2.4E-3 and T is given by

$$T = \frac{\tau_b - \tau_{cs}}{\tau_{cs}} = \frac{u_*^2 - u_{*cs}^2}{u_{*cs}^2} \tag{5.33}$$

where τ_b is the bed stress and τ_{cs} is the critical Shields stress. The use of Smith and McLean's formulation requires that the critical Shields stress be specified for each sediment size class. Van Rijn's formula is

$$S_{eq} = 0.015\rho_s \frac{d}{z_{eq}^*} T^{3/2} R_d^{-1/5} \tag{5.34}$$

where z_{eq}^* ($= Hz_{eq}$) is the dimensional reference height and R_d is a sediment grain Reynolds number

$$R_d = \left(g\left(\frac{\rho_s}{\rho} - 1\right) d \right)^{1/2} \frac{d}{\nu} \tag{5.35}$$

When Van Rijn's formula is select for use in EFDC, the critical Shields stress in internally calculated using relationships from Van Rijn (1984). Van Rijn suggested setting the dimensional reference height to three grain diameters. In the EFDC model, the user specifies the reference height as a multiple of the largest noncohesive sediment size class diameter.

Garcia and Parker's general formula for multiple sediment size classes is

$$S_{jeq} = \rho_s \frac{A(\lambda Z_j)^5}{\left(1 + 3.33A(\lambda Z)^5\right)} \tag{5.36}$$

$$Z_j = \frac{u_*}{w_{sj}} R_{dj}^{3/5} F_H \tag{5.37}$$

$$F_H = \left(\frac{d_j}{d_{50}}\right)^{1/5} \tag{5.38}$$

$$\lambda = 1 + \frac{\sigma_\phi}{\sigma_{\phi o}}(\lambda_o - 1) \tag{5.39}$$

where A is a constant equal to 1.3E-7, d_{50} is the median grain diameter based on all sediment classes, λ is a straining factor, F_H is a hiding factor and σ_ϕ is the standard deviation of the sedimentological phi scale of sediment size distribution. Garcia and Parker's formulation is unique in that it can account for armoring effects when multiple sediment classes are simulated. For simulation of a single noncohesive size class, the straining factor and the hiding factor are set to one. The EFDC model has the option to simulate armoring with Garcia and Parker's formulation. For armoring simulation, the current surface layer of the sediment bed is restricted to a thickness equal to the dimensional reference height.

6. Cohesive Sediment Settling, Deposition and Resuspension

The settling of cohesive inorganic sediment and organic particulate material is an extremely complex process. Inherent in the process of gravitational settling is the process of flocculation, where individual cohesive sediment particles and particulate organic particles aggregate to form larger groupings or flocs having settling characteristics significantly different from those of the component particles (Burban et al., 1989,1990; Gibbs, 1985; Mehta et al., 1989). Floc formation is dependent upon the type and concentration of the suspended material, the ionic characteristics of the environment, and the fluid shear and turbulence intensity of the flow environment. Progress has been made in first principles mathematical modeling of floc formation or aggregation, and disaggregation by intense flow shear (Lick and Lick, 1988; Tsai, et al., 1987). However, the computational intensity of such approaches precludes direct simulation of flocculation in operational cohesive sediment transport models for the immediate future.

An alternative approach, which has met with reasonable success, is the parameterization of the settling velocity of flocs in terms of cohesive and organic material fundamental particle size, d; concentration, S; and flow characteristics such as vertical shear of the horizontal velocity, du/dz, shear stress, $A_v du/dz$, or turbulence intensity in the water column or near the sediment bed, q. This has allowed semi-empirical expressions having the functional form

$$W_{se} = W_{se}\left(d, S, \frac{du}{dz}, q \right) \qquad (6.1)$$

to be developed to represent the effective settling velocity. A widely used empirical expression, first incorporated into a numerical by Ariathurai and Krone (1976), relates the effective settling velocity to the sediment concentration:

$$w_s = w_{so}\left(\frac{S}{S_o} \right)^a \qquad (6.2)$$

with the o superscript denoting reference values. Depending upon the reference concentration and the value of α, this equation predicts either increasing or decreasing settling velocity as the sediment concentration increases. Equation (6.2) with user defined base settling velocity, concentration and exponent is an option in the EFDC model. Hwang and Metha (1989) proposed

$$w_s = \frac{aS^n}{\left(S^2 + b^2 \right)^m} \qquad (6.3)$$

based on observations of settling at six sites in Lake Okeechobee. This equation has a general parabolic shape with the settling velocity decreasing with decreasing concentration at low concentrations and decreasing with increasing concentration at high concentration. A least squares for the paramters, a, m, and n, in (6.3) was shown to agree well with observational data. Equation (6.3) does not hav a dependence on flow characteristics, but is based on data from an energetic field condition having both currents and high frequency surface waves. A generalized form of (6.3) can be selected as an option in the EFDC model.

Ziegler and Nisbet, (1994, 1995) proposed a formulation to express the effective settling as a function of the floc diameter, d_f

$$w_s = a d_f^b \qquad (6.4)$$

with the floc diameter given by:

$$d_f = \left(\frac{\alpha_f}{S\sqrt{\tau_{xz}^2 + \tau_{xz}^2}} \right)^{1/2} \qquad (6.5)$$

where S is the sediment concentration, α_f is an experimentally determined constant and τ_{xz} and τ_{yz} are the x and y components of the turbulent shear stresses at a given position in the water column. Other quantities in (6.4) have been experimentally determined to fit the relationships:

$$a = B_1\left(S\sqrt{\tau_{xz}^2 + \tau_{xz}^2}\right)^{-0.85} \tag{6.6}$$

$$b = -0.8 - 0.5\log\left(S\sqrt{\tau_{xz}^2 + \tau_{xz}^2} - B_2\right) \tag{6.7}$$

where B_1 and B_2 are experimental constants. This formulation is also an option in the EFDC model.

A final settling option in EFDC is based on that proposed by Shrestha and Orlob (1996). The formulation in EFDC has the form

$$w_s = S^{\alpha}\exp(-4.21 + 0.147G) \tag{6.8}$$
$$\alpha = 0.11 + 0.039G$$

where

$$G = \sqrt{(\partial_z u)^2 + (\partial_z v)^2} \tag{6.9}$$

is the magnitude of the vertical shear of the horizontal velocity. It is noted that all of these formulations are based on specific dimensional units for input parameters and predicted settling velocities and that appropriate unit conversion are made internally in their implementation in the EFDC model.

Water column-sediment bed exchange of cohesive sediments and organic solids is controlled by the near bed flow environment and the geomechanics of the deposited bed. Net deposition to the bed occurs as the flow-induced bed surface stress decreases. The most widely used expression for the depositional flux is:

$$J_o^d = \begin{cases} -w_s S_d\left(\dfrac{\tau_{cd} - \tau_b}{\tau_{cd}}\right) = -w_s T_d S_d & : \quad \tau_b \le \tau_{cd} \\ 0 & : \quad \tau_b \ge \tau_{cd} \end{cases} \tag{6.10}$$

where τ_b is the stress exerted by the flow on the bed, τ_{cd} is a critical stress for deposition which depends on sediment material and floc physiochemical properties (Mehta et al., 1989) and S_d is the near bed depositing sediment concentration. The critical deposition stress is generally determined from laboratory or in situ field observations and values ranging form 0.06 to 1.1 N/m**2 have been reported in the literature. Given this wide range of reported values, in the absence of site specific data the depositional stress and is generally treated as a calibration parameter. The depositional stress is an input parameter in the EFDC model.

Since the near bed depositing sediment concentration in (6.10) is not directly calculated, the procedures of Chapter 5 can be applied to relate the the near bed depositional concentration to the bottom layer or depth averge concentration. Using (5.14) the near bed concentration during times of deposition can be determined in terms of the bottom layer concentration for three-dimensional model applications. Inserting (6.10) into (5.14) and evaluating the constant at a near bed depositional level gives

$$S = \left(T_d + (1 - T_d) \frac{z_d^R}{z^R} \right) S_d \tag{6.11}$$

Integrating (6.11) over the bottom layer gives

$$\begin{aligned} S_d &= \left(T_d + \frac{\ln\left(\Delta z_d^{-1}\right)}{\left(\Delta z_d^{-1} - 1\right)} (1 - T_d) \right)^{-1} \bar{S} : R = 1 \\ S_d &= \left(T_d + \frac{\left(\left(\Delta z_{eq}^{-1}\right)^{1-R} - 1\right)}{(1 - R)\left(\Delta z_d^{-1} - 1\right)} (1 - T_d) \right)^{-1} \bar{S} : R \neq 1 \end{aligned} \tag{6.12}$$

The corresponding quantities in the numerical solution bottom boundary condition (3.6) are

$$\begin{aligned} P_d w_s &= \left(T_d + \frac{\ln\left(\Delta z_d^{-1}\right)}{\left(\Delta z_d^{-1} - 1\right)} (1 - T_d) \right)^{-1} w_s : R = 1 \\ P_d w_s &= \left(T_d + \frac{\left(\left(\Delta z_{eq}^{-1}\right)^{1-R} - 1\right)}{(1 - R)\left(\Delta z_d^{-1} - 1\right)} (1 - T_d) \right)^{-1} w_s : R \neq 1 \end{aligned} \tag{6.13}$$

For depth averaged model application, (6.10) is combined with (5.25) and the constant of integration is evaluated at a near bed depositional level to give

$$S = \left(1 - \frac{Rz}{(1 + R)} \right) T_d S_d + \left(1 - \left(1 - \frac{Rz_d}{(1 + R)} \right) T_d \right) S_d \frac{z_d^R}{z^R} \tag{6.14}$$

Integrating (6.14) over the depth gives

$$\begin{aligned} S_d &= \left(\left(\frac{2 + R(1 - z_d)}{2(1 + R)} \right) T_d + \frac{\ln\left(z_d^{-1}\right)}{\left(z_d^{-1} - 1\right)} \left(1 - \left(\frac{1 + R(1 - z_d)}{(1 + R)} \right) T_d \right) \right)^{-1} \bar{S} : R = 1 \\ S_d &= \left(\left(\frac{2 + R(1 - z_d)}{2(1 + R)} \right) T_d + \frac{\left(z_d^{R-1} - 1\right)}{(1 - R)\left(z_d^{-1} - 1\right)} \left(1 - \left(1 - \frac{Rz_d}{(1 + R)} \right) T_d \right) \right)^{-1} \bar{S} : R \neq 1 \end{aligned} \tag{6.15}$$

The corresponding quantities in the numerical solution bottom boundary condition (3.6) are

$$P_d w_s = \left(\left(\frac{2 + R(1 - z_d)}{2(1 + R)} \right) T_d + \frac{\ln(z_d^{-1})}{(z_d^{-1} - 1)} \left(1 - \left(\frac{1 + R(1 - z_d)}{(1 + R)} \right) T_d \right) \right)^{-1} w_s : R = 1$$

$$P_d w_s = \left(\left(\frac{2 + R(1 - z_d)}{2(1 + R)} \right) T_d + \frac{(z_d^{R-1} - 1)}{(1 - R)(z_d^{-1} - 1)} \left(1 - \left(1 - \frac{R z_d}{(1 + R)} \right) T_d \right) \right)^{-1} w_s : R \neq 1 \quad (6.16)$$

It is noted that the assumptions used to arrive at the relationships, (6.12) and (6.15) are more teneous for cohesive sediment than the similar relationships for noncohesive sediment. The settling velocity for cohesive sediment is highly concentration dependent and the use of a constant settling velocity to arrive at (6.12) and (6.15) is questionable. The specification of an appropriate reference level for cohesive sediment is difficult. One possibility is to relate the reference level to the floc diameter using (6.5). An alternative is to set the reference level to a laminar sublayer thickness

$$z_d = \frac{\nu(S)}{H u_*} \quad (6.17)$$

where $\nu(S)$ is a sediment concentration dependent kinematic viscosity and the water depth is include to nondimensionlize the reference level. A number of investigators, including Mehta and Jiang (1990) have presented experimental results indicating that at high sediment concentrations, cohesive sediment-water mixtures behave as high viscosity fluids. Mehta and Jain's results indicate that a sediment concentration of 10,000 mg/L results in a viscosity ten time that of pure water and that the viscosity increases logrithmically with increasing mixture density. Use of the relationships (6.12) and (6.16) is optional in the EFDC model. When they are used, the reference height is set using (6.17) with the viscosity determined using Mehta and Jain's experimental relationship between viscosity and sediment concentration. To more fully address the deposition prediction problem, a nested sediment, current and wave boundary layer model based on the near bed closure presented in Chapter 4 is under development.

Cohesive bed erosion occurs in two distinct modes, mass erosion and surface erosion. Mass erosion occurs rapidly when the bed stress exerted by the flow exceeds the depth varying shear strength, τ_s, of the bed at a depth, H_{me}, below the bed surface. Surface erosion occurs gradually when the flow-exerted bed stress is less than the bed shear strength near the surface but greater than a critical erosion or resuspension stress, τ_{ce}, which is dependent on the shear strength and density of the bed. A typical scenario under conditions of accelerating flow and increasing bed stress would involve first the occurrence of gradual surface erosion, followed by a rapid interval of mass erosion, followed by another interval of surface erosion. Alternately, if the bed is well consolidated with a sufficiently high shear strength profile, only gradual surface erosion would occur. Transport into the water column by mass or bulk erosion can be expressed in the form

$$J_o^r = w_r S_r = \frac{m_{me}(\tau_s \le \tau_b)}{T_{me}} \tag{6.18}$$

where J_o is the erosion flux, the product $w_r S_r$ represents the numerical boundary condition (3.6), m_{me} is the dry sediment mass per unit area of the bed having a shear strength, τ_s, less than the flow-induced bed stress, τ_b, and T_{me} is a somewhat arbitrary time scale for the bulk mass transfer. The time scale can be taken as the numerical model integration time step (Shrestha and Orlob, 1996). Observations by Hwang and Mehta (1989) have indicated that the maximum rate of mass erosion is on the order of 0.6 gm/s-m**2 which provides an means of estimating the transfer time scale in (4.10). The shear strenght of the cohesive sediment bed is generally agreed to be a linear function of the bed bulk density (Metha et al., 1982; Villaret and Paulic, 1986; Hwang and Mehta, 1989)

$$\tau_s = a_s \rho_b + b_s \tag{6.19}$$

For the shear strength in N/m**2 and the bulk density in gm/cm**3, Hwang and Mehta (1989) give a_s and b_s values of 9.808 and -9.934 for bulk density greater than 1.065 gm/cm**3. The EFDC model currently implements Hwang and Mehta's relationship, but can be readily modified to incorporated other functional relationships.

Surface erosion is generally represented by relationships of the form

$$J_o^r = w_r S_r = \frac{dm_e}{dt}\left(\frac{\tau_b - \tau_{ce}}{\tau_{ce}}\right)^{\alpha} \quad : \quad \tau_b \ge \tau_{ce} \tag{6.20}$$

or

$$J_o^r = w_r S_r = \frac{dm_e}{dt}\exp\left(-\beta\left(\frac{\tau_b - \tau_{ce}}{\tau_{ce}}\right)^{\gamma}\right) \quad : \quad \tau_b \ge \tau_{ce} \tag{6.21}$$

where dm_e/dt is the surface erosion rate per unit surface area of the bed and τ_{ce} is the critical stress for surface erosion or resuspension. The critical erosion rate and stress and the parameters α, β, and γ are generally determined from laboratory or in situ field experimental observations. Equation (6.20) is more appropriate for consolidated beds, while (6.21) is appropriate for soft partially consolidated beds. The base erosion rate and the critical stress for erosion depend upon the type of sediment, the bed water content, total salt content, ionic species in the water, pH and temperature (Mehta, *et al*., 1989) and can be measured in laboratory and sea bed flumes.

The critical erosion stress is related to but generally less than the shear strength of the bed, which in turn depends upon the sediment type and the state of consolidation of the bed. Experimentally determined relationships between the critical surface erosion stress and the dry density of the bed of the form

$$\tau_{ce} = c\rho_s^d \tag{6.22}$$

have been presented (Mehta, et al., 1989). Hwang and Mehta (1989) proposed the relationship

$$\tau_{ce} = a(\rho_b - \rho_l)^b + c \tag{6.23}$$

between the critical surface erosion stress and the bed bulk density with *a, b, c*, and ρl equal to 0.883, 0.2, 0.05, and 1.065, respectively for the stress in N/m**2 and the bulk density in gm/cm**3. Considering the relationship between dry and bulk density

$$\rho_d = \rho_s \frac{(\rho_b - \rho_w)}{(\rho_s - \rho_w)} \tag{6.24}$$

equations (6.22) and (6.23) are consistent. The EFDC model allow for a user defined constant critial stress for surface erosion or the use of (6.23). Alternate predictive expression can be readily incorporated into the model.

Surface erosion rates ranging from 0.005 to 0.1 gm/s-m**2 have been reported in the literature, and it is generally accepted that the surface erosion rate decreases with increasing bulk density. Based on experimental observations, Hwang and Mehta (1989) proposed the relationship

$$\log_{10}\left(\frac{dm_e}{dt}\right) = 0.23\exp\left(\frac{0.198}{\rho_b - 1.0023}\right) \tag{6.25}$$

for the erosion rate in mg/hr-cm**2 and the bulk density in gm/cm**3. The EFDC model allow for a user defined constant surface erosion rate or predicts the rate using (6.25). Alternate predictive expression can be readily incorporated into the model. The use of bulk density functions to predict bed strength and erosion rates in turn requires the prediction of time and depth in bed variations in bulk density which is related to the water and sediment density and the bed void ratio by

$$\rho_b = \left(\frac{\varepsilon}{1+\varepsilon}\right)\rho_w + \left(\frac{1}{1+\varepsilon}\right)\rho_s \tag{6.26}$$

Selection of the bulk density dependent formulations in the EFDC model requires implmentation of a bed consolidation simulation to predict the bed void ratio as discussed in the following chapter.

7. Sediment Bed Geomechanical Processes

This chapter describes the representation of the sediment bed in the EFDC model. To make the information presented self contained, the derivation of mass balance equations and comparison with formulations used in other models is also presented.

Consider a sediment bed represented by discrete layers of thickness B_k, which may be time varying. The conservation of sediment and water mass per unit horizontal area in layer k is given by:

$$\partial_t\left(\frac{\rho_s B_k}{1+\varepsilon_k}\right) = J_{s:k-} - J_{s:k+} \tag{7.1}$$

$$\partial_t\left(\frac{\rho_w \varepsilon_k B_k}{1+\varepsilon_k}\right) = J_{w:k-} - J_{w:k+} \tag{7.2}$$

where ε is the void ratio, ρ_s and ρ_w are the sediment and water density and J_s and J_w are the sediment and water mass flux with k- and k+ defining the bottom and top boundaries, respectively of layer k. The mass flux is define as positive in the vertical direction. Assuming sediment and water to be incompressible (7.1) and (7.2) can be written as:

$$\partial_t\left(\frac{B_k}{1+\varepsilon_k}\right) = \frac{J_{s:k-}}{\rho_s} - \frac{J_{s:k+}}{\rho_s} \tag{7.3}$$

$$\partial_t\left(\frac{\varepsilon_k B_k}{1+\varepsilon_k}\right) = \frac{J_{w:k-}}{\rho_w} - \frac{J_{w:k+}}{\rho_w} \tag{7.4}$$

For the bed layer, $k=k_a$, adjacent to the water column the sediment and water flux are:

$$J_{s:ka+} = J_{sb} \tag{7.5}$$

$$J_{w:ka+} = \rho_w q_{ka+} + \varepsilon_{ka}\frac{\rho_w}{\rho_s}\max(J_{sb},0) + \varepsilon_b\frac{\rho_w}{\rho_s}\min(J_{sb},0) \tag{7.6}$$

where J_{sb} is the net sediment mass flux from the bed to the water column, q_w is the specific discharge of water due to bed consolidation, and ε_b is the water column void ratio at the water column-sediment bed interface. The last two terms in (7.6) represent entrainment of bed water into the water column during sediment resuspension and entrainment of water column water into the bed during deposition, respectively. At bed layer interfaces not adjacent to the water column, the water flux is:

$$\begin{aligned} J_{w:k+} &= \rho_w q_{k+} \quad : \quad k \neq k_a \\ J_{w:k-} &= \rho_w q_{k-} \end{aligned} \tag{7.7}$$

The sediment flux at bed layer interfaces not adjacent to the water column is either specified or determined in the course of solving the sediment mass conservation equation. Combining equations (7.1) and (7.5) and equations (7.2), (7.6), and (7.7) gives the mass conservation equations

$$\partial_t \left(\frac{\rho_s B_{ka}}{1+\varepsilon_{ka}} \right) = J_{s:ka-} - J_{sb} \tag{7.8}$$

$$\partial_t \left(\frac{\rho_w \varepsilon_{ka} B_{ka}}{1+\varepsilon_{ka}} \right) = \rho_w \left(q_{ka-} - q_{ka+} \right) - \frac{\rho_w}{\rho_s} \left(\varepsilon_{ka} \max\left(J_{sb}, 0\right) + \varepsilon_b \min\left(J_{sb}, 0\right) \right) \tag{7.9}$$

for the bed layer adjacent to the water column. Restating (7.1) and combining equations (7.2) and (7.7) gives the mass conservation equations

$$\partial_t \left(\frac{\rho_s B_k}{1+\varepsilon_k} \right) = J_{s:k-} - J_{s:k+} \tag{7.10}$$

$$\partial_t \left(\frac{\rho_w \varepsilon_k B_k}{1+\varepsilon_k} \right) = \rho_w \left(q_{k-} - q_{k+} \right) \tag{7.11}$$

for layers not adjacent to the water column.

Four approaches for the solution of the mass conservation equations (7.8) through (7.11) have been previously utilized. The solution approaches, hereafter referred to as solution levels, increase in complexity and physical realism and will be briefly summarized. The first level or simplest approach assumes specified time-constant layer thicknesses and void ratios with the left sides of (7.8) through (7.11) being identically zero. Sediment mass flux at all layer interfaces are then identical to the net flux from the bed to the water column. Bed representations at this level, as exemplified by the RECOVERY model (Boyer, et al., 1994), typically omit the water mass conservation equations. However, it is noted that the water mass conservation is ill posed unless either q_{1-}, the specific discharge at the bottom of the deepest layer or q_{ka+}, the specific discharge at the top of the water column adjacent layer, is specified. If q_{1-} is set to zero, q_{ka+} is then required to exactly cancel the entrainment terms is (7.9).

The second level of bed mass conservation representation assumes specified time invariant layer thicknesses. The sediment mass conservation equations (7.8) and (7.10) for the water column adjacent layer and the underlying layers become

$$B_{ka} \partial_t S_{ka} = J_{s:ka-} - J_{sb} \tag{7.12}$$

$$B_k \partial_t S_k = J_{s:k-} - J_{s:k+} \tag{7.13}$$

where

$$S_k = \frac{\rho_s}{1+\varepsilon_k} \tag{7.14}$$

is the sediment concentration (mass of sediment per volume of sediment-water mixture). Considering a two layer bed (k_a = 2) and noting that $J_{1+} = J_{2-}$, readily reveals that the formulation is ill posed unless the internal sediment flux is specified or appropriately paramterized. A parameterization which is employed for the constant bed layer thickness option in the WASP5 model (Ambrose, et al., 1993) is

$$\begin{aligned} J_{s:k-} &= -w_{b:k-} S_k \\ J_{s:k+} &= -w_{b:k+} S_{k+1} \\ w_{b:k+} &= w_{b:k+1-} \end{aligned} \tag{7.15}$$

where w_b is a specified burial velocity. The mass balance equations (7.12) and (7.13) using (7.15) are properly posed and can be solved for the sediment concentration. In the event that the sediment concentration in the water column adjacent layer becomes negative, the layer is eliminated and the underlying layer become water column adjacent. The solution of (7.12) and (7.13) for the sediment concentration then allows the void ratio to be determined from (7.14). The left sides of the water mass conservation equations (7.9) and (7.11) are thus known and these equations are more appropriately written as

$$q_{ka+} - q_{ka-} = -B_{ka}\partial_t\left(\frac{\varepsilon_{ka}}{1+\varepsilon_{ka}}\right) - \frac{1}{\rho_s}\left(\varepsilon_{ka}\max\left(J_{sb},0\right) + \varepsilon_b \min\left(J_{sb},0\right)\right) \tag{7.16}$$

$$q_{k+} - q_{k-} = -B_k\partial_t\left(\frac{\varepsilon_{ka}}{1+\varepsilon_{ka}}\right) \tag{7.17}$$

The determination of the specific discharges using (7.16) and (7.17) can be viewed is either ill posed or physically inconsistent. As shown for the first level approach, the solution of (7.16) and (7.17) is ill posed unless either q_{1-}, the specific discharge at the bottom of the deepest layer or q_{ka+}, the specific discharge at the top of the water column adjacent layer must be independently specified. If q_{1-} is specified and the internal specific discharges determined from (7.17), q_{ka+} is then required to partially cancel the entrainment terms is (7.16). As will be subsequently shown, the specific discharges can be dynamically determined using Darcy's law formulated in terms of void ratio. However, the specific discharges determined using Darcy's law and the known void ratios are not guaranteed to satisfy (7.16) and (7.17) and the level two formulation is dynamically inconsistent with respect to water mass conservation in the sediment bed. The constant bed layer thickness option in the WASP5 ignores this problem entirely by not considering the water mass balance and hence neglecting pore water advection of dissolved contaminants.

The third level of bed mass conservation representation assumes specified time invariant layer void ratios. The sediment mass conservation equations (7.8) and (7.10) for the water column adjacent layer and the underlying layers become

$$S_{ka}\partial_t B_{ka} = J_{s:ka-} - J_{sb} \tag{7.18}$$

$$S_k \partial_t B_k = J_{s:k-} - J_{s:k+} \tag{7.19}$$

where the sediment concentration S_k defined by (7.14) is time invariant. This level requires independent specification of the bottom sediment flux, $J_{s:ka-}$, in the water column adjacent layer which allows (7.18) to be solved for the time varying water column adjacent layer thickness. The solution of (7.19) then requires specification of either the layer bottom sediment flux $J_{s:k-}$ or dB_k/dt in underlying layers. The water mass conservation equations (7.9) and (7.11) for this level of representation are:

$$q_{ka+} - q_{ka-} = -\left(\frac{\varepsilon_{ka}}{1+\varepsilon_{ka}}\right)\partial_t B_{ka} - \frac{1}{\rho_s}\left(\varepsilon_{ka}\max\left(J_{sb},0\right) + \varepsilon_b \min\left(J_{sb},0\right)\right) \tag{7.20}$$

$$q_{k-} - q_{k+} = \left(\frac{\varepsilon_k}{1+\varepsilon_k}\right)\partial_t B_k \tag{7.21}$$

where the right sides are known form the solution of the sediment mass conservation equations. The solution of (7.20) requires specification of either the bottom or top specific discharges. Subsequently (7.21) can be solved successively downward for the layer bottom specific discharges.

The variable bed layer thickness option in the WASP5 model (Ambrose, et al., 1993) exemplifies the third level of bed representation. Specifically, the thickness of the water column adjacent layer is allowed to vary in time, while the thicknesses of the underlying layers remain constant, with equations (7.18) and (7.19) becoming

$$S_{ka}\partial_t B_{ka} = J_{s:ka-} - J_{sb} \tag{7.22}$$

$$J_{s:k-} = J_{s:k+} \tag{7.23}$$

The periodic time variation specified for the water column adjacent layer bottom flux is:

$$\begin{aligned} J_{s:ka-} &= 0 \quad : \quad t_o \le t \le t_o + (N-1)\Delta t \\ J_{s:ka-} &= \int_{t_o}^{t_o+N\Delta t} J_{sb}\,dt \quad : \quad t_o + (N-1)\Delta t \le t \le t_o + N\Delta t \end{aligned} \tag{7.24}$$

where Δt is the standard water time step and $N\Delta t$ is the sediment compaction time. This results in the thickness of the water column adjacent layer periodically returning to its initial value at time intervals of $N\Delta t$ unless the thickness becomes negative due to net resuspension. In that event, the underlying layer becomes the water column adjacent layer. The water mass conservation (7.21) for layers not adjacent to the water column becomes

$$q_{k+} = q_{k-} = q_{1-} \quad : \quad k \neq k_a \tag{7.25}$$

indicating that all internal specific discharges are equal a specified specific discharge at the bottom of layer 1. Given the solution for the time variation of the water column adjacent thickness and bottom specific discharge, (7.20) can be solved for the specific discharge at the top of the layer.

The constant porosity bed option in EFDC is also a level three approach. In EFDC, the internal sediment fluxes are set to zero and the change in thickness of the water column adjacent layer is determined directly using (7.18) while the underlying layers have time invariant thicknesses. As a result, the internal water specific discharges are set to zero and the water entrainment and expulsion in the water column adjacent layer are determined directly from (7.20). The EFDC model is configured to have a user specified maximum number of sediment bed layer. A the start of a simulation, the number of layers containing sediment at a specific horizontal location is specified. Under continued deposition, a new water column layer is created when the thickness of the current layer exceeds a user specified value. If the current water column adjacent layer's index is equal to the maximam number of layers, the bottom two layers are combined and the remaining layers renumbered before addition of the new layer. Under continued resuspension, the layer underlying the current water column adjacent layer becomes the new adjacent layer when all sediment is resuspended form the current layer.

The fourth level of bed representation accounts for bed consolidation by allowing the layer void ratios and thicknesses to vary in time. The simplest and most elegant formulations at this level utilize a Lagrangian approach for sediment mass conservation in layers not adjacent to the water column. The Lagrangian approach requires that the sediment mass per unit horizontal area be time invariant and without loss of generality the internal sediment fluxes are set to zero in layers not adjacent to the bed. Consistent with these requirements, (7.8) and (7.10) become

$$\partial_t \left(\frac{\rho_s B_{ka}}{1+\varepsilon_{ka}} \right) = -J_{sb} \tag{7.26}$$

$$\partial_t \left(\frac{\rho_s B_k}{1+\varepsilon_k} \right) = 0 \quad : \quad k \neq k_a \tag{7.27}$$

These equations are readily integrated to give

$$\left(\frac{\rho_s B_{ka}}{1+\varepsilon_{ka}} \right)^{n+1} = \left(\frac{\rho_s B_{ka}}{1+\varepsilon_{ka}} \right)^{n} - \Delta_t J_{sb}^{n+1/2} \tag{7.28}$$

$$\left(\frac{\rho_s B_k}{1+\varepsilon_k}\right)^{n+1} = \left(\frac{\rho_s B_k}{1+\varepsilon_k}\right)^{n} \quad : \quad k \neq k_a \tag{7.29}$$

where *n* and *n+1* denote the old and new times, and Δt is the time interval between *n* and *n+1*. The Lagrangian approach for sediment mass conservation also requires that the number of bed layers vary in time. Under conditions of continued deposition, a new water column adjacent layer would be added when either the thickness, void ratio or mass per unit area of the current water column adjacent layer reaches a predefined value. Under conditions of continued resuspension, the bed layer immediately under the current water column adjacent layer would become the new water column adjacent layer when the entire sediment mass of the current layer has been resuspended. The water mass conservation equations remain unchanged from their general forms give by (7.9) and (7.10) which for convenience are restated as

$$\partial_t\left(\frac{\rho_w \varepsilon_{ka} B_{ka}}{1+\varepsilon_{ka}}\right) = \rho_w\left(q_{ka-} - q_{ka+}\right) - \frac{\rho_w}{\rho_s}\left(\varepsilon_{ka}\max\left(J_{sb},0\right) + \varepsilon_b \min\left(J_{sb},0\right)\right) \tag{7.30}$$

$$\partial_t\left(\frac{\rho_w \varepsilon_k B_k}{1+\varepsilon_k}\right) = \rho_w\left(q_{k-} - q_{k+}\right) \quad : \quad k \neq k_a \tag{7.31}$$

At the fourth and most realistic level of bed representation, three approaches can be used to represent bed consolidation. Two of the approaches are semi-empirical with the first assuming that the void ratio of a layer decreases with time. A typical relationship which is used for the simple consolidation option in the EFDC model is

$$\varepsilon = \varepsilon_m + \left(\varepsilon_o - \varepsilon_m\right)\exp\left(-\alpha\left(t - t_o\right)\right) \tag{7.32}$$

where ε_o is the void ratio at the mean time of deposition, t_o, ε_m is the ultimate minimum void ratio corresponding to complete consolidation, and α is an empirical or experimental constant. Use of (7.32) in the EFDC model involves specifying the depositional and ultimate void ratios and the rate constant. The actual calculation involves using the initial void ratios to determine the deposition time t_o, after which (7.32) is used to update the void ratios as the simulation progresses. After equation (7.32) is used to calculate the new time level void ratios, the results of equations (7.28) and (7.29) provide the new layer thicknesses. The water conservation equations (7.30) and (7.31) can then be solved using

$$q_{ka+}^{n+1/2} = q_{ka-}^{n+1/2} - \frac{1}{\Delta_t}\left(\frac{\varepsilon_k B_k}{1+\varepsilon_k}\right)^{n+1} + \frac{1}{\Delta_t}\left(\frac{\varepsilon_k B_k}{1+\varepsilon_k}\right)^{n} - \frac{1}{\rho_s}\left(\varepsilon_{ka}\max\left(J_{sb},0\right) + \varepsilon_b \min\left(J_{sb},0\right)\right)^{n+1/2} \tag{7.33}$$

$$q_{k+}^{n+1/2} = q_{k-}^{n+1/2} - \frac{1}{\Delta_t}\left(\frac{\varepsilon_k B_k}{1+\varepsilon_k}\right)^{n+1} + \frac{1}{\Delta_t}\left(\frac{\varepsilon_k B_k}{1+\varepsilon_k}\right)^{n} \quad : \quad k \neq k_a \tag{7.34}$$

to determine the water specific discharges, provided that the specific discharge q_{1-}, at the bottom of layer *1* is specified. When this option is specified in the EFDC model, the specific discharge at bottom of the bottom sediment layer is set to zero. Layers are added and deleted in the manner previously described for EFDC's constant porosity option.

The second semi-empirical approach assumes that the vertical distribution of the bed bulk density or equivalently the, void ratio at any time is given by a self-similar function of vertical position, bed thickness and fixed surface and bottom bulk densities or void ratios. Functionally this equivalent to

$$\varepsilon = V(z, B_T, \varepsilon_{ka}, \varepsilon_1) \tag{7.35}$$

where V represents the function, z is a vertical coordinate measured upward from the bottom of the lowest layer, and B_T is the total thickness of the bed. This approach is used in the original HSTM model (Hayter and Mehta, 1983), the new HSCTM model (Hayter et al., 1998) and is an option in the CE-QUAL-ICM/TOXI model (Dortch, et al., 1998). This approach also appears to be employed in the SED2D-WES model (Letter et al., 1998) and related models (Shrestha and Orlob, 1996). The determination of the new time level layer thicknesses and void ratios requires an iterative solution of equations (7.28), (7.29) and (7.35). The solution is completed using (7.33) and (7.34) to determine the water specific discharges.

The third and most realistic approach is to dynamically simulate the consolidation of the bed. The consolidation equations are derived by expanding the left sides of (7.30) and (7.31) giving

$$\begin{aligned}\left(\frac{B_{ka}}{1+\varepsilon_{ka}}\right)\partial_t \varepsilon_{ka} + \varepsilon_{ka}\partial_t\left(\frac{B_{ka}}{1+\varepsilon_{ka}}\right) &= q_{ka-} - q_{ka+} \\ &-\frac{1}{\rho_s}\left(\varepsilon_{ka}\max(J_{sb},0) + \varepsilon_b \min(J_{sb},0)\right)\end{aligned} \tag{7.36}$$

$$\left(\frac{B_{ka}}{1+\varepsilon_k}\right)\partial_t \varepsilon_k + \varepsilon_k \partial_t\left(\frac{B_k}{1+\varepsilon_{ka}}\right) = q_{k-} - q_{k+} \quad : \quad k \neq k_a \tag{7.37}$$

Using (7.26) and (7.27), equations (7.36) and (7.37) become

$$\left(\frac{B_{ka}}{1+\varepsilon_{ka}}\right)\partial_t \varepsilon_{ka} = q_{ka-} - q_{ka+} + \frac{1}{\rho_s}(\varepsilon_{ka} - \varepsilon_b)\min\left(\frac{J_{sb}}{\rho_s}, 0\right) \tag{7.38}$$

$$\left(\frac{B_{ka}}{1+\varepsilon_k}\right)\partial_t \varepsilon_k = q_{k-} - q_{k+} \quad : \quad k \neq k_a \tag{7.39}$$

The specific discharges in (7.38) and (7.39) are determined using the Darcy equation

$$q = -\frac{K}{g\rho_w}\partial_z u \tag{7.40}$$

where K is the hydraulic conductivity and u is the excess pore pressure defined as the difference between the total pore pressure u_t, and the hydrostatic pressure u_h.

$$u = u_t - u_h \tag{7.41}$$

The total pore pressure is defined as the difference between the total stress σ and effective stress σ_e.

$$u_t = \sigma - \sigma_e \tag{7.42}$$

The total stress and hydrostatic pressure are given by

$$\sigma = p_b + g\left(\left(\frac{\varepsilon}{1+\varepsilon}\right)\rho_w + \left(\frac{1}{1+\varepsilon}\right)\rho_s\right)(z_b - z) \tag{7.43}$$

$$u_h = p_b + g\rho_w(z_b - z) \tag{7.44}$$

where p_b is the water column pressure at the bed z_b. Solving for the excess pore pressure using (7.41) through (7.44) gives

$$u = g\rho_w\left(\frac{\rho_s}{\rho_w} - 1\right)\left(\frac{1}{1+\varepsilon}\right)(z_b - z) - \sigma_e \tag{7.45}$$

which is introduced into (7.40) to give

$$q = \frac{K}{g\rho_w}\partial_z \sigma_e + \left(\frac{\rho_s}{\rho_w} - 1\right)\left(\frac{K}{1+\varepsilon}\right) \tag{7.46}$$

or

$$q = \frac{K}{g\rho_w}\left(\frac{d\sigma_e}{d\varepsilon}\right)\partial_z \varepsilon + \left(\frac{\rho_s}{\rho_w} - 1\right)\left(\frac{K}{1+\varepsilon}\right) \tag{7.47}$$

where $d\varepsilon/d\sigma_c$ is a coefficient of compressibility.

For consistency with the Lagrangian representation of sediment mass conservation, a new vertical coordinate ζ, defined by

$$\frac{d\zeta}{dz} = \frac{1}{1+\varepsilon} \tag{7.48}$$

is introduced. The discrete form of (7.48) is

$$\zeta_{k+} - \zeta_{k-} = \frac{z_{k+} - z_{k-}}{1+\varepsilon_k} = \frac{B_k}{1+\varepsilon_k} = \Delta_{\zeta:k} \tag{7.49}$$

Introducing (7.48) into (7.47) gives

$$q = \lambda\left(\frac{K}{1+\varepsilon}\right)\partial_\zeta \varepsilon + \left(\frac{\rho_s}{\rho_w} - 1\right)\left(\frac{K}{1+\varepsilon}\right) \tag{7.50}$$

where

$$\lambda = \frac{1}{g\rho_w}\frac{d\sigma_e}{d\varepsilon} \tag{7.51}$$

is a compressibility length. Introducing (7.49) and (7.50) into (7.38) and (7.39) gives

$$\begin{aligned} &\Delta_{\zeta:ka}\partial_t \varepsilon_{ka} + \left(\frac{\rho_s}{\rho_w} - 1\right)\left(\left(\frac{K}{1+\varepsilon}\right)_{ka+} - \left(\frac{K}{1+\varepsilon}\right)_{ka-}\right) \\ &+ \left(\lambda\left(\frac{K}{1+\varepsilon}\right)\partial_\zeta \varepsilon\right)_{ka+} - \left(\lambda\left(\frac{K}{1+\varepsilon}\right)\partial_\zeta \varepsilon\right)_{ka-} = (\varepsilon_{ka} - \varepsilon_b)\min\left(\frac{J_{sb}}{\rho_s}, 0\right) \end{aligned} \tag{7.52}$$

for the surface adjacent layer and

$$\begin{aligned} &\Delta_{\zeta:k}\partial_t \varepsilon_k + \left(\frac{\rho_s}{\rho_w} - 1\right)\left(\left(\frac{K}{1+\varepsilon}\right)_{k+} - \left(\frac{K}{1+\varepsilon}\right)_{k-}\right) \\ &+ \left(\lambda\left(\frac{K}{1+\varepsilon}\right)\partial_\zeta \varepsilon\right)_{k+} - \left(\lambda\left(\frac{K}{1+\varepsilon}\right)\partial_\zeta \varepsilon\right)_{k-} = 0 \end{aligned} \tag{7.53}$$

for remaining layers.

Equation (7.53) is the discrete form of the finite strain consolidation equation

$$\partial_t \varepsilon + \left(\frac{\rho_s}{\rho_w} - 1\right)\partial_\zeta\left(\frac{K}{1+\varepsilon}\right) + \partial_\zeta\left(\frac{1}{g\rho_w}\frac{\partial\sigma_e}{\partial\varepsilon}\left(\frac{K}{1+\varepsilon}\right)\partial_\zeta \varepsilon\right) = 0 \tag{7.54}$$

first derived by Gibson et al. (1967). Since

$$\frac{\partial \sigma_e}{\partial \varepsilon} \leq 0 \tag{7.55}$$

(7.54) is formally a parabolic or diffusion equation. Equation (7.54) was used by Cargill (1985) in the formulation of a model for dredge material consolidation and by Le Normant (1998) to represent bed consolidation in a three-dimensional cohesive sediment transport model. The classic linear consolidation equation (Middleton and Wilcock, 1994) omits the second term associated with self weight in (7.54) and introduces a constant consolidation coefficient

$$C_c = -(1+\varepsilon)\frac{\partial \sigma_e}{\partial e}\frac{K}{g\rho_w} \tag{7.56}$$

reducing (7.54) to

$$\partial_t \varepsilon = C_c \partial_{zz} \varepsilon \tag{7.57}$$

Equation (7.57) has separable solutions of the form

$$\begin{aligned} \varepsilon &= \phi_n(\zeta)\exp\left(-\lambda_n \frac{C_c}{B^2} t\right) \\ \partial_{\zeta\zeta}\phi_n + \lambda_n \phi_n &= 0 \\ \zeta &= \frac{z}{B} \end{aligned} \tag{7.58}$$

which provides some justification for empirical relationship (7.32). The solution of the finite strain consolidation equations, (7.52) and (7.53), requires constitutive relationships

$$\begin{aligned} \frac{K}{1+\varepsilon} &= f_1(\varepsilon) \\ \frac{\partial \sigma_e}{\partial \varepsilon} &= f_2(\varepsilon) \end{aligned} \tag{7.59}$$

Cargill (1985) presents graphical forms of these relationships based on laboratory analysis of cohesive sediments from a number of estuaries. The finite strain consolidation formulation is currently being implement in the EFDC and will be tested and released upon selection of a generic set of constituitive relationship.

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